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Contents

Abstract

In this paper the basic theory for a big hydrogene balloon with sails - Andrées polar balloon, the "Örnen"[1] – is lined out.

The force equation for the "Örnen" is derived in chapter 1, detailed enough to make a foundation for a simulation program[2].
The vertical component of this equation and its physical application is by no means trivial, but it is its horisontal components that have caused confusion, debate and controverses. It seems impossible to deviate from the wind direction with a vehicle that drifts with the wind and that is of course quite right. For a balloon that drifts with a constant wind, no relative wind (V_A) can be felt on the balloon. In other words: Ground speed (V_G) = Wind speed (V_W) so $V_A = V_W - V_G = 0$. Then a sail would hang loose, regardless of wind speed - the balloon travels in the wind direction with $V_G = V_W$.
Now suppose that we hang down a rope that drags on the ground. The balloon will then be braked to a lower V_G, which means that we will get some V_A. Then we get wind in the sail and then the sail produces a force. Now – by rotating the balloon (and thereby the sail) around its vertical axis, we can make the sail force to deviate from the wind direction. Then the total force on the balloon deviates from the wind direction and drives the whole vehicle in that new direction. The balloon accelerates until the sail force plus air force on the balloon+rig just balances the drag rope force.

An absolute condition for the generation of this side force is that the vehicle can be rotated in order to get the sails in the wanted angle relative to the wind and that it can be kept there. This rotation is achieved by moving the fastening point of the drag rope along the periphery of the basket. The big question now is "will the moment from the drag rope balance the moment from the sail force so that we can keep the sail at the wanted angle relative to the wind"? This question cannot be answered simply by reasoning - it requires some mathematical analysis, involving the geometry of the device, see chapter 2 below.
Like any other mathematical analysis of a physics problem, it rests on a simplified picture of the physical system. In this case the most daring simplifications are
a) the aerodynamics of the sail is not deteriorated by the presence of the balloon
b) the vehicle is a rigid body.
Taken resonable uncertainties into account, the analysis still shows that it is possible to keep the balloon rotated to an angle within a certain envelope by use of the drag rope.

The simulation rests on the assumption that the vehicle is moment wise stable and trimmable. In chapter 2 it is shown that this condition is/was fulfilled in what can be assumed to be the intended flight envelope.

[1] Swedish for "The Eagle"
[2] www.kueng.se/Balloon-H2/

1. The force equation

1.1 Total force, acting on vehicle, $\bar{F}^E_{tot}$

$$\bar{F}^E_{tot} = \frac{d\left(\dot{\bar{r}}^E_{BE} \bullet m_{tot}\right)}{dt} = \ddot{\bar{r}}^E_{BE} \cdot m_{tot} + \dot{\bar{r}}^E_{BE} \cdot \dot{m}_{tot} \approx \ddot{\bar{r}}^E_{BE} \cdot m_{tot}$$

where

$$\begin{cases} \bar{F}^E_{tot} = \bar{F}^E_{sail} + \bar{F}^E_{rope} + \bar{F}^E_{Adyn} + \bar{F}^E_{Astat} + \bar{F}^E_{g} \quad \text{(see the sections below)} \\ \dot{m}_{tot} = \dfrac{d\left[m_{B0} + \left(L_{tot} - L_{rog}\right)\lambda_{rope} + \rho_G V_G\right]}{dt} = -\dfrac{dL_{rog}}{dt}\lambda_{rope} + \dfrac{\delta\rho_G}{\delta r^E_{zBE}}\dot{r}^E_{zBE}V_G + \rho_G\dot{V}_G \end{cases}$$

The indicated approximation is good – the term $\dot{\bar{r}}^E_{BE} \cdot \dot{m}_{tot}$ is neglectable for all practical cases.
The different parts that build up the force $\bar{F}^E_{tot}$ are derived below.

1.2 Wind and ”relative wind”

$\dot{\bar{r}}^E_{AE} = \begin{pmatrix} V_w & 0 & 0 \end{pmatrix}'$ speed of air (A) ($\equiv$ wind speed) relative to ground (E), expressed in S_E.

$\dot{\bar{r}}_{AB} = \dot{\bar{r}}_{AE} + \dot{\bar{r}}_{EB} = \dot{\bar{r}}_{AE} - \dot{\bar{r}}_{BE}$ speed of air (A) relative to vehicle (B), "relative wind".

1.3 Aerodynamic force due to sail, $\bar{F}^E_{sail}$

The horizontal component of the ”lift” force from the sail is perpendicular to $\dot{\bar{r}}_{AB}$ - it has the direction $\pm$ norm($\dot{\bar{r}}_{AB} \times \hat{z}_B$), where the sign shall be plus for sail to the left and minus for sail to the right. The vertical component (due to inclination of the sail) has the direction $\hat{z}_B$.
The drag from the sail has the direction norm($\dot{\bar{r}}_{AB}$).
Thereof

$$\bar{F}^B_{sail} = \pm C_{L_{sail}} q_{xy} S_{ref} \bullet \text{norm}(\dot{\bar{r}}^B_{AB} \times \hat{z}^B_B) + C_{z_{sail}} \cdot \hat{z}^B_B + C_{D_{sail}} q_{xy} S_{ref} \bullet \text{norm}(\dot{\bar{r}}^B_{AB})$$

and, since $M_{BE} = E$

$$\bar{F}^E_{sail} = \pm C_{L_{sail}} q_{xy} S_{ref} \bullet \text{norm}(\dot{\bar{r}}^B_{AB} \times \hat{z}^B_B) + C_{z_{sail}} \cdot \hat{z}^B_B + C_{D_{sail}} q_{xy} S_{ref} \bullet \text{norm}(\dot{\bar{r}}^B_{AB})$$

Aero data for $\alpha = 90^\circ$ and $\alpha = \pm 60^\circ$, see Diagram 3.1, are used in the simulation model.

1.4 Force due to drag rope friction, $\bar{F}^E_{rope}$

Note: This is about that part of the force that comes from the friction – the mass of the free rope is included in m_B, see section 1.8 below.
The horisontal component of the force from the drag rope is counter directed to the ground speed of the balloon – its direction is $\left(-\hat{x}_T\right)$. The vertical component has the direction $\hat{z}_E$.
Thereof

$$\bar{F}^E_{rope} = -L_{rog}\lambda_{rope} g\mu \cdot \left(M_{ET}\hat{x}^T_T + tg\left(\varepsilon\right) \cdot \hat{z}^E_E\right)$$

where ε is the angle between the horizontal plane and the straight line that connects the ends of the freely hanging part of the rope, see appendix 2.

1.5 Aerodynamic force on vehicle exept sail, $\bar{F}^E_{Adyn}$

The direction of the air drag is norm($\dot{\bar{r}}_{AB}$).
Thereof
$\bar{F}^E_{Adyn} = C_D q S_{ref} \cdot norm(\dot{\bar{r}}^B_{AB})$
where the relation $M_{BE} = E$ has been used.
Note: Here, as for the sail force above, the interference between balloon and sail is neglected.

1.6 Aerostatic force, $\bar{F}^E_{Astat}$

The aerostatic force – the float force – is calculated as the weight of the displaced air mass.
It's direction is $(-\hat{z}_E)$
Consequently:
$\bar{F}^E_{Astat} = -\rho_A V_G g \cdot \hat{z}_E$
The density ρ_A, as well as static pressure P_A and temperature T_A of ambient air is determined as a function of altitude from a standard atmosphere[3].
The equation of state for an ideal gas says $\frac{P}{\rho R} = T$. Assuming $T_G = T_A$ gives[4]

$$\frac{P_A}{\rho_A R_A} = \frac{P_G}{\rho_G R_G}$$

A safety valve keeps $P_A \leq P_G \leq P_A + \Delta P_{GA}$ by letting out gas when needed:

$$\Delta P_G = \frac{\Delta m_G}{V_{G\max}} T_G R_G$$

"Book-keeping" of the remaining mass of contained gas makes it possible to determine the gas volume, needed in eq :
$V_G = m_G \rho_G$
where ρ_G is determined by eq .
Most of the flight time $V_G < V_{G\max} \Rightarrow P_G = P_A$. Then eq is reduced to

$$\rho_G = \rho_A \frac{R_A}{R_G}$$

3 U.S.Standard Atmosphere 1962, see an application in http://www.kueng.se/Atmos/
4 The "excuse" for this simplification is that calculations show that a moderate temperature difference (T_A-T_G) has very little – if any - influence on control and flying qualities of the balloon and therefore can be excluded in a dynamic model of the vehicle. Seen from other aspects (like endurance) it does matter, though:
The lift change due to temperature difference ($T_G > T_A$) at H < 1000 can be calculated to ca $+15[N/{}^\circ C]$.
This means that if T_G is increased by $1[{}^\circ C]$ (with $P_G = P_A$ and T_A = constant), the lift force is increased by ca $15[N]$ (due to loss of gas at constant volume). Sunshine from a clear sky could cause T_A to rise $20[{}^\circ C]$ in 40 minutes (see ref. 2), which means a considerable lift increase ($300[N]$) at the cost of ditto gas loss ($30[kg]$).

1.7 The transformation matrix M_{ET}

M_{ET} is needed for calculation of $\bar{F}^E_{rope}$, see above. It can be established as

$M_{ET} = \left(\hat{x}^E_T \quad \hat{y}^E_T \quad \hat{z}^E_T \right)'$ Note the transpose!

where

$$\begin{cases} \hat{x}^E_T = norm(\dot{\bar{r}}^E_{BE}) \\ \hat{z}^E_T = \hat{z}^E_E \\ \hat{y}^E_T = \hat{z}^E_E \times \hat{x}^E_T \end{cases}$$

1.8 Mass, m_{tot}

$$m_{tot} = m_B + m_G$$

where

$$m_B = m_{B0} + (L_{tot} - L_{rog})\lambda_{rope}$$

$$m_G = \rho_G V_G$$

1.9 Gravitation force, $\bar{F}^E_g$

$$\bar{F}^E_g = m_{tot}\bar{g}^E$$

2. The moment equation

2.1 Problem formulation

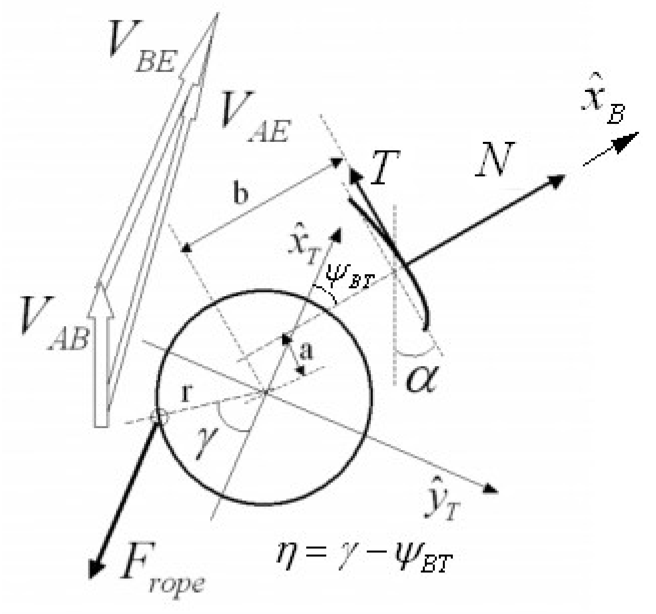


Figure 2.1.1

With conventional sign rules, the stability criterion is formulated as

$$\begin{cases} \text{Static yaw stability} \Rightarrow \quad C^B_{mz\alpha} > 0 \\ \text{Dynamic yaw stability} \Rightarrow \quad C^B_{mz\dot{\alpha}} > 0 \end{cases}$$

The system is trimmable if $C_{mz} = 0$ can be achieved and maintained (possibly as center value of an oscillation).

Due to the symmetry with respect to the $\hat{x}_B \hat{z}_B$-plane, only wind from the right side ($90^\circ \le \alpha \le 0^\circ$) is delt with.
It will be shown that the device is trimmable and stable for all relevant angles of atack.

The moment in $\hat{z}_B$ direction:

$$\left.\begin{aligned} M_z &= -T \cdot b + N \cdot a - F_{rope} \cdot r \sin\gamma + M_{z0} + M_{zd} \\ M_z &= I_z \cdot \dot{\omega}^B_{zBE} \approx I_z \cdot \ddot{\alpha} \end{aligned}\right\}$$

where

$$\left.\begin{aligned}
&N = q_A \cdot S_{sail} \cdot C_{N_{sail}} \\
&T = q_A \cdot S_{sail} \cdot C_{T_{sail}} \\
&F_{rope} = q_A \cdot S_{sail} \cdot \left(C_{D_{balloon}} + C_{T_{sail}} \sin\psi_{BT} + C_{N_{sail}} \cos\psi_{BT} \right) \\
&\text{Force eq. in } \hat{y}_T \text{ direction gives } \psi_{BT} = \arctan\left(\frac{T}{N}\right) = \arctan\left(\frac{C_{T_{sail}}}{C_{N_{sail}}}\right) \\
&M_{zd} = f\left(\omega_{zBE}, V_{AB}\right) \text{, see appendix 1.}
\end{aligned}\right\}$$

The moment in normalized shape:

$$\left.\begin{aligned}
&C_{mz} = -C_{T_{sail}} \cdot \frac{b}{c_{sail}} + C_{N_{sail}} \cdot \frac{a}{c_{sail}} - C_{F\,rope} \cdot \frac{r}{c_{sail}} \sin\gamma + C_{mz0} + C_{mzd} \\
&\text{where} \\
&C_{Frope} = C_{Dballoon} + C_{Tsail} \cdot \sin\psi_{BT} + C_{Nsail} \cdot \cos\psi_{BT}
\end{aligned}\right\}$$

2.2 Trim

Trim in this case means $C_{mz} = 0$ and $\omega^{B}_{zBE} = 0 \; (\Rightarrow \; M_{zd} = 0)$.
This gives

$$0 = -C_{T_{sail}} \cdot \frac{b}{c_{sail}} + C_{N_{sail}} \cdot \frac{a}{c_{sail}} - C_{F\,rope} \cdot \frac{r}{c_{sail}} \sin\gamma + C_{mz0} + C_{mzd}$$

$$\gamma_{trim} = \arcsin\left[\left(-C_{T_{sail}} \cdot \frac{b}{c_{sail}} + C_{N_{sail}} \cdot \frac{a}{c_{sail}} + C_{mz0} + C_{mzd}\right) \cdot \left(C_{F\,rope} \cdot \frac{r}{c_{sail}}\right)^{-1}\right]$$

The angle γ is referred to the flight path direction $\hat{x}_T$. It seems more practical to use the angle η, which is referred to the body axis $\hat{x}_B$:

$$\eta_{trim} = \gamma_{trim} - \psi_{BT}$$

η is the angular displacement of the rope, needed to rotate the balloon to get the trim value $\alpha = \alpha_{trim}$ (from sails hanging symmetrically up front, $\alpha = 90^\circ$).

$\eta_{trim} = f(\alpha)$ and $\gamma_{trim} = f(\alpha)$ can be seen in Diagram 1.

2.3 Stability

$$C_{mz} = -C_{T_{sail}} \cdot \frac{b}{c_{sail}} + C_{N_{sail}} \cdot \frac{a}{c_{sail}} - C_{F\,rope} \cdot \frac{r}{c_{sail}} \sin\gamma + C_{mz0} + C_{mzd}$$

Derivation with respect to α :

$$C_{mz\alpha} = -C_{T\alpha_{sail}} \cdot \frac{b}{c_{sail}} + C_{N\alpha_{sail}} \cdot \frac{a}{c_{sail}} - \frac{r}{c_{sail}} \cdot \frac{\partial}{\partial\alpha}\left(C_{F\,rope} \cdot \sin\gamma \right)$$

where

$$\frac{\partial}{\partial\alpha}\left(C_{F\,rope} \cdot \sin\gamma \right) = \left(C_{T\alpha_{sail}} \sin\psi_{BT} + C_{N\alpha_{sail}} \cos\psi_{BT} \right) \cdot \sin\gamma - C_{F\,rope} \cdot \cos\gamma$$

The period time and damping can be calculated from

$$C_{m\alpha} \cdot \alpha \cdot q_a \cdot S_{ref} \cdot c_{ref} + \frac{\partial M_{zdamp}}{\partial \omega_z} \cdot \dot{\alpha} = -I_z \cdot \ddot{\alpha}$$

For $\frac{\partial M_{zdamp}}{\partial \omega_z}$, see appendix 1.

$$\ddot{\alpha} + 2q\dot{\alpha} + h^2\alpha = 0$$

$$q = \frac{\partial M_{zdamp}}{\partial \omega_z} \cdot \frac{1}{2I_z}$$

$$h^2 = C_{m\alpha} \cdot \frac{q_a \cdot S_{ref} \cdot c_{ref}}{I_z}$$

From the solution of we get the damped period time and the time to half amplitude:

$$T_d = 2\pi \cdot \left(h^2 - q^2 \right)^{-0.5}$$

$$T_{½} = \frac{\ln 2}{q}$$

Diagram 2.3.1 shows stability and trim data, calculated by use of data in Diagram 2.4.1.

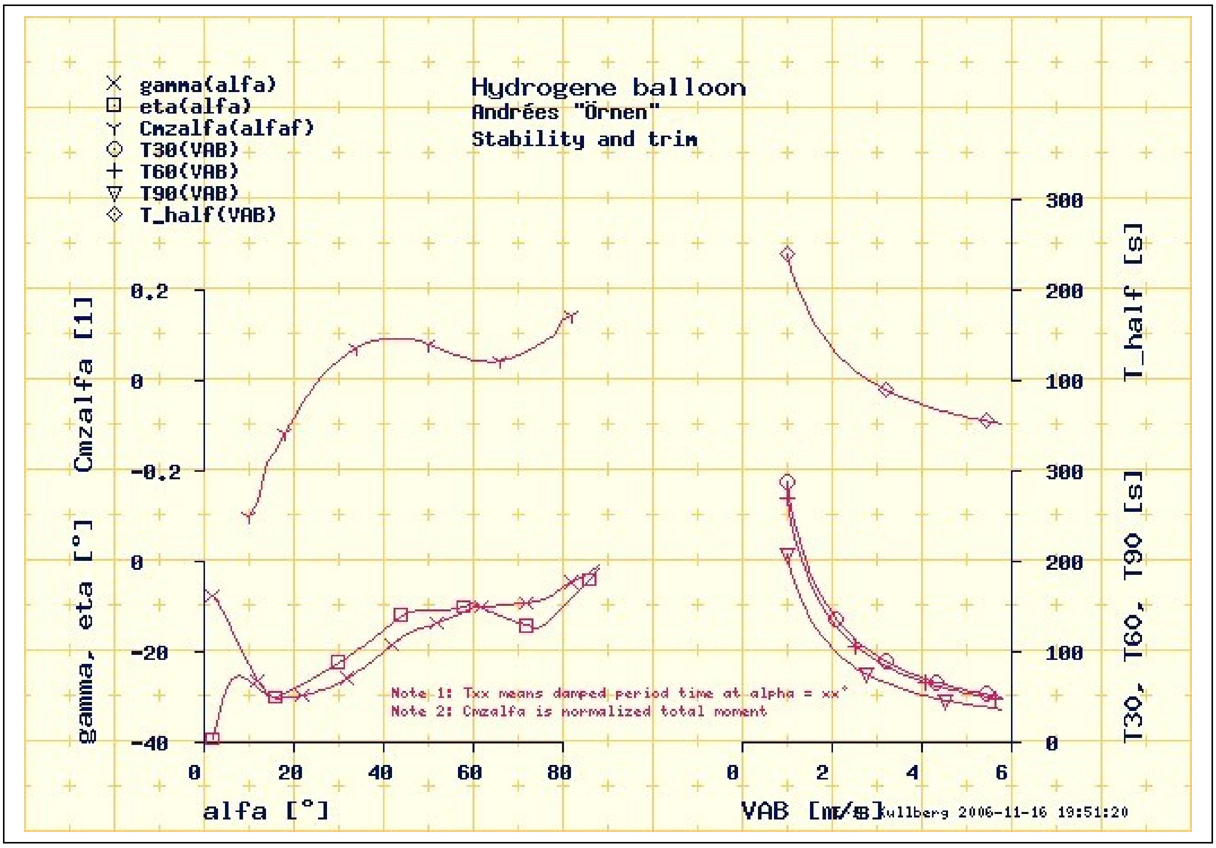


Diagram 2.3.1

The $C_{m\alpha}(\alpha)$ -curve shows that we have static yaw stability for $\alpha > 25^\circ$.

The yaw damping, here expressed as $T_{½}$ (time to half amplitude) was calculated using the result in appendix 1.

3. *Aero data*

Diagram 3.1 shows $\left(C_N \quad C_T \quad tc \quad a\right) = f\left(\alpha\right)$, where a is defined in fig. 2.1.1. Data are symmetrical around $\alpha = 90^\circ$ (= sail up front).

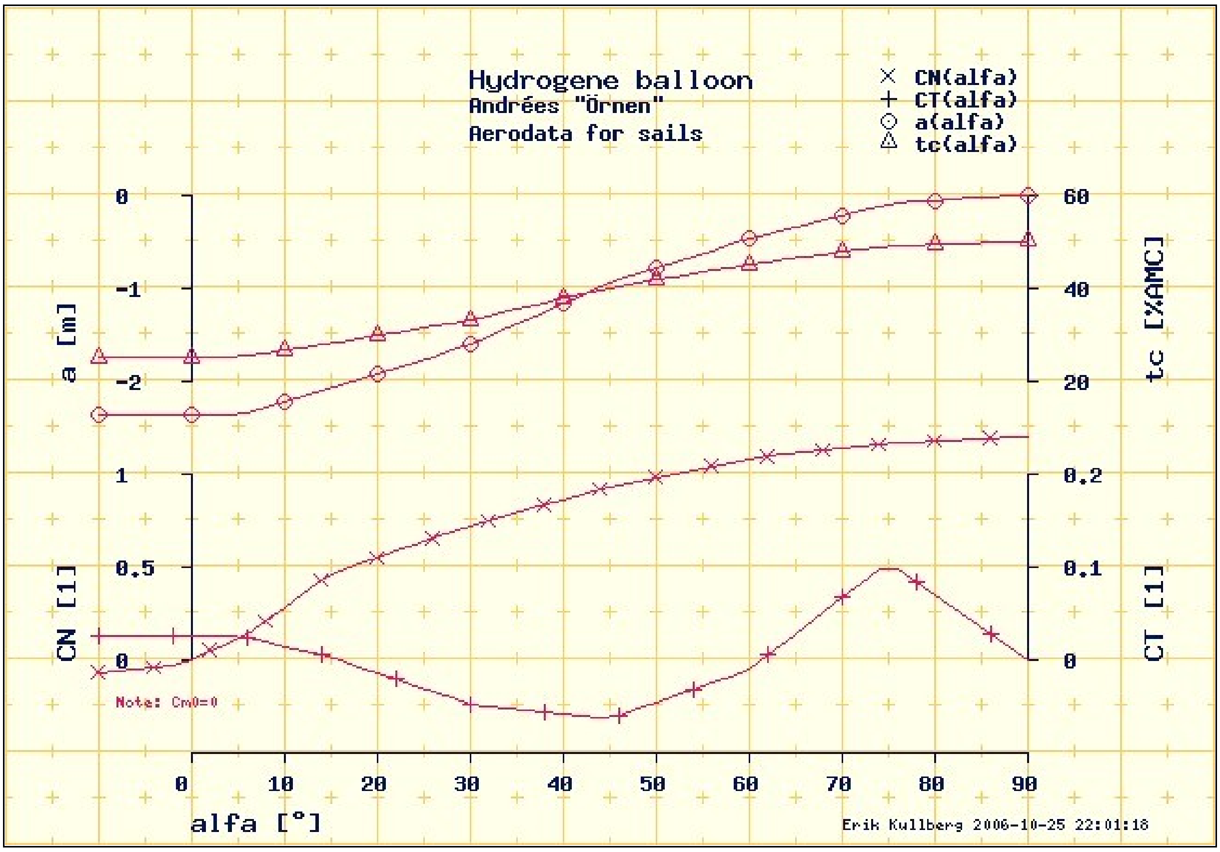
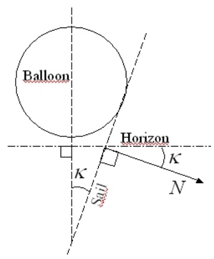


Diagram 3.1

The influence from the balloon on the sails has not been taken into account – the sails are supposed to work in undisturbed air.

The sail is inclined an angle $\kappa = 35.4^\circ$ relative to the vertical, see fig. to the right. The aero data in Diagram 2.4.1 are expressed in a system where the xz-plane coincides with the sail plane. Let's transform to a system with horizontal/vertical axis:



$$C_T = C_T^D$$

$$C_N = C_N^D \cdot \cos\kappa$$

$$C_Z = C_N^D \cdot \sin\kappa$$

where upper index D marks data from Diagram 3.1.

In some cases – like section 1.4 above – the aerodynamic coefficients are wanted in wind oriented system instead of body oriented. The transformation is:

$$\begin{pmatrix} C_{Dsail} \\ C_{Lsail} \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} C_T \\ C_N \end{pmatrix}$$

C_z is not affected by this transformation: $C_{zsail} = C_z$.

4. *Symbols and definitions*

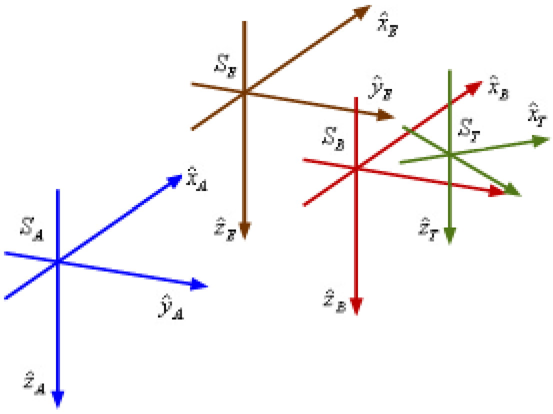


Figure 1.1.1

General:
All coordinate systems are orthogonal right hand systems, all vectors are column vectors.
S_Q is a coordinate system with origin in $\bar{r}_Q$ and axis directions $\hat{x}_Q$, $\hat{y}_Q$ and $\hat{z}_Q$.

$\hat{S}_Q$ means "axis directions of S_Q" or the matrix $\begin{pmatrix} \hat{x}'_Q \\ \hat{y}'_Q \\ \hat{z}'_Q \end{pmatrix}$.

$\bar{r}^C_{AB}$ is to be read "r dash, A relative B, in C" and interpreted as "position vector for (point) A relative (point) B, expressed in (system) S_C".

M_{QR} is a transformation matrix. Its index order defines the transformation direction in the following manner: $\bar{r}^Q_P = M_{QR}\bar{r}^R_P$

Arend specific symbols and definitions:
S_E is earth fixed regarding origo position $\bar{r}_E$ and axis directions.
S_B is vehicle fixed, $\bar{r}_B$ is position of the vehicles center of gravity.
S_A is air fixed, $\bar{r}_A$ is position of an air molecule A.
$\hat{S}_B = \hat{S}_E = \hat{S}_A \Rightarrow M_{BE} = M_{EA} = E$ (E is unit matrix)

S_T is vehicle fixed, $\begin{cases} \bar{r}_T = \bar{r}_B \rightarrow \bar{r}_{TB} = \bar{0} & \text{(In the figure } \bar{r}_{TB} \neq \bar{0} \text{ for clarity)} \\ \hat{z}_T = \hat{z}_B \\ \hat{x}_T = norm(\dot{\bar{r}}_{BE}) \end{cases}$

ρ_A ... density of ambient air
ρ_G ... density of in the balloon contained gas
P_A ... static pressure of ambient air
P_G ... static pressure of in the balloon contained gas
ΔP_{GA} pressure differens, $\Delta P_{GA} = P_G - P_A$

μ ... friction coefficient for rope on ground (gravel, water, ice etc)

q ... dynamic pressure of the vehicle, $q = \frac{\rho_A}{2}\left|\dot{\bar{r}}_{BA}\right|^2$

T_A ... Absolute temperature of ambient air

T_G ... Absolute temperature of enclosed gas

R_A ... Gas constant for air

R_G ... Gas constant for the enclosed gas

C_D ... drag coefficient of vehicle exept sail (assumed independent of relative wind direction)

$C_{L_{sail}}$ ... Horizontal component of "lift"-coefficient of the sail.

$C_{D_{sail}}$ ... drag coefficient of the sail.

C_{Zsail} ... Vertical component of "lift"-coefficient of the sail.

V_w ... wind speed (relative ground), same as V_{AB}

L_{rog} ... length of rope on ground

L_{rout} ... length of rope outside the basket

L_{tot} ... total rope length

m_{B0} ... mass of vehicle, excluding rope and contained gas.

m_G ... mass of in the balloon contained gas

m_B ... mass of vehicle including airborne rope, excluding contained gas.

m_{tot} ... total mass of vehicle, $m_{tot} = m_B + m_G$

λ_{rope} ... mass per length unit for drag rope

g ... earth acceleration, $\bar{g}^E = \begin{pmatrix}0 & 0 & 9.81\end{pmatrix}'$

V_G ... volume of contained gas

ε ... angle between rope and ground plane, the rope assumed straight.

tc ... pressure center position

$V_{BE} \equiv \dot{r}_{BE}$ Velocity of balloon rel. to earth

$V_{AB} \equiv \dot{r}_{AB}$ Velocity of air rel. to balloon,

$V_{AE} \equiv \dot{r}_{AE}$ Velocity of air rel. to earth

α ... angle of attack for sail

I_z ... moment of inertia around z_B axis (for the whole vehicle, regarded as a rigid body)

ω_{zBE} ... the z component of the angular velocity $\bar{\omega}_{BE}$

ψ_{BT} ... euler yaw angle ψ for S_B rel. to S_T

T_d ... damped period time

$T_{½}$ ... time to half amplitude

5. References

1. S.A. Andrée, The beginning of polar aviation 1895-1897
   Günter Sollinger
   ISBN 5-900078-42-7

2. Teknisk Tidskrft 10 jan 1931, “Planerade Nordpolsexpeditioner ... “ , Tord Ångström

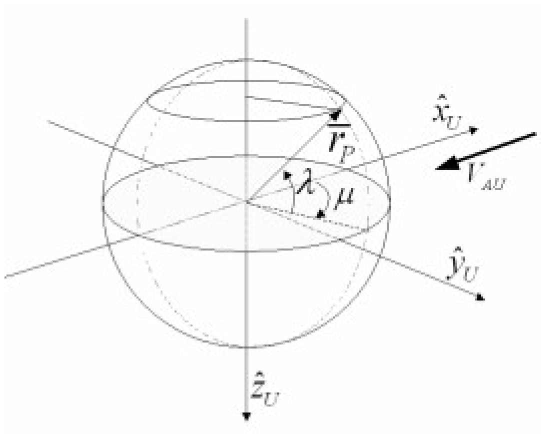
6. Acknowledgement

The aero data $\left(C_N \quad C_T \quad tc\right) = f\left(\alpha\right)$, see chapter 3, were calculated and made available by mr Kenneth Nilsson, M.Sc. He also served as a testpilot/guineapig during the testing of the simulation program and contributed with many valuable suggestions.

7. Appendix 1: *Aerodynamic damping of a slowly rotating sphere*

The aerodynamic damping of the yaw oscillation of a spheric balloon will be derived below. The damping moment is supposed to come from friction between balloon skin and air. In reality also the friction on the inside (skin/gas) and the cross motion of the drag rope contribute, but here only the outer skin friction part will be analyzed.
The solution is purely geometric – it does not take pressure distribution into account. This would not be an acceptable method for an object in high relative wind, rotating with high angular rate – like a golf ball – but it is deemed acceptable for the normally very slow rotation and low relative winds of a manned balloon.

Assume the balloon to be a sphere, see figure to the right.
S_B is rotating relative S_U with the angular velocity $\bar{\omega}_{BU}$.
The vector $\bar{r}_P^U = r_P \cdot \left(\cos\lambda \, \cos\mu \quad \cos\lambda \, \sin\mu \quad -\sin\lambda \right)$ defines a sphere that coincides with the balloon, but does not rotate.



The wind in the point $\bar{r}_P$ is built up by two components - one caused by the rotation $\bar{\omega}_{BE}^U$ of the balloon and one caused by the relative wind V_{AU}.

The undisturbed relative wind velocity $\dot{\bar{r}}_{AU}$ causes a local wind in $\bar{r}_P$, close to the balloon surface, that is perpendicular to $\bar{r}_{PU}$ and lies in the $\hat{r}_{PU}\hat{x}_U$ -plane[5]. Its direction is $\hat{\dot{r}}_{AU} = -\hat{x}_P$.
The direction $\hat{x}_P$ is defined by

$$\hat{S}_P^U = \begin{pmatrix} \hat{x}_P^U = \hat{y}_P^U \times \hat{z}_P^U \\ \hat{y}_P^U = \hat{z}_P^U \times \hat{x}_U^U \\ \hat{z}_P^U = -\hat{r}_{PU}^U \end{pmatrix}$$

The airspeed at the point $\bar{r}_P$ is derived from the position of an air molecule relative to S_P:

$$\bar{r}_{PA}^U = \bar{r}_{PB}^U + \bar{r}_{BA}^U = M_{UB}\bar{r}_{PB}^B + \bar{r}_{BA}^U$$

$$\dot{\bar{r}}_{PA}^U = \dot{M}_{UB}\bar{r}_{PB}^B + M_{UB}\dot{\bar{r}}_{PB}^B + \dot{\bar{r}}_{BA}^U$$

$$\left.\begin{array}{l} \dot{M}_{UB}\bar{r}_{PB}^B = M_{UB}\tilde{\omega}_{BU}^B\bar{r}_{PB}^U = \bar{\omega}_{BU}^U \times \bar{r}_{PB}^U \\ M_{UB}\dot{\bar{r}}_{PB}^B = \dot{\bar{r}}_{BP}^U = 0 \\ \dot{\bar{r}}_{BA}^U\left(\bar{r}_{PB}^U\right) \approx -V_{AU}\hat{x}_P^U \end{array}\right\}$$

$$\dot{\bar{r}}_{PA}^U = \bar{\omega}_{BU}^U \times \bar{r}_{PB}^U - V_{AU}\hat{x}_P^U$$

The friction force between balloon shell and air due to relative wind in point $\bar{r}_P$ can now be written as

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[5]The influence from the basket and the support ropes is neglected; the flow is assumed to be rotational symmetric around $\hat{x}_U$. Furthermore – the streamline pattern is supposed to be symmetrical with respct to the $\hat{y}_U\hat{z}_U$ -plane. These deviations from reality can to some extent be compensated for in the choice of friction coefficient value.

$$d\bar{F}_P^U = C_F \cdot \frac{\rho \left|\dot{\bar{r}}_{PA}^U\right|^2}{2} d\lambda \cdot d\mu \cdot r_{PU}^2 \cdot norm\left(\dot{\bar{r}}_{PA}^U\right)$$

The moment in $\bar{r}_U$ due to $d\bar{F}_P$ is

$$d\bar{M}_U^U = \bar{r}_{PU}^U \times d\bar{F}_P^U$$

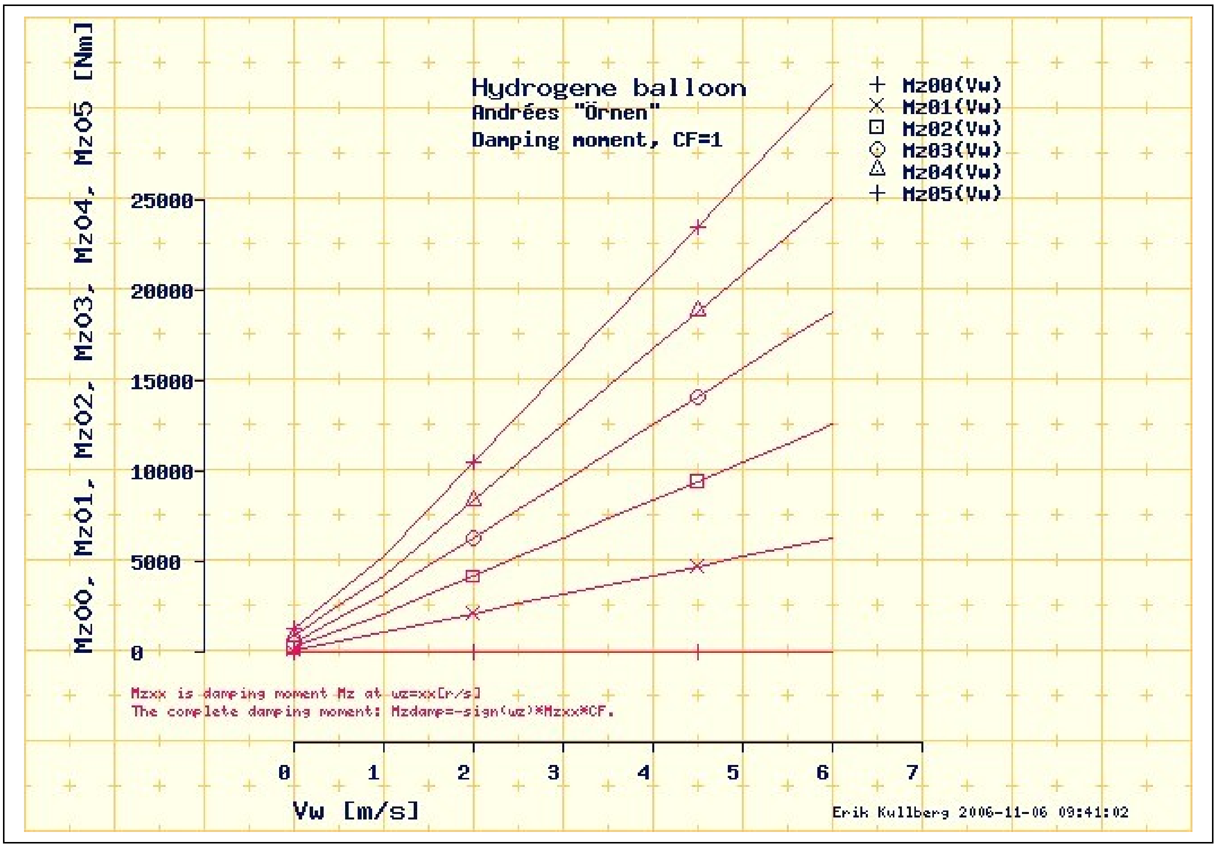
The total moment due to friction:

$$\bar{M}_U^U = C_F \cdot \frac{\rho\, r_{PU}^2}{2} \iint \left|\dot{\bar{r}}_{PA}^U\right|^2 \cdot norm\left(\dot{\bar{r}}_{PA}^U\right) d\lambda \cdot d\mu$$

This was calculated numerically and it´s z-component was tabelled (for $M_z \geq 0$) as

$$M_{ztab} = f\left(\left|\omega_z\right| \quad V_{AU}\right)$$

This tabelled function is plotted below, using $C_F = 1, \rho = 1.225[\mathrm{kg/m^3}], r_{PU} = 10[\mathrm{m}]$.



The complete damping moment is calculated as

$$M_{zdamp} = -sign\left(\omega_z\right) \cdot M_{ztab} \cdot C_F$$

where M_{ztab} is the interpolated value from the tabelled function.

Simulations show that $0.005 \leq C_F \leq 0.01$ give reasonable results. This is then supposed to include all the different damping effects on the balloon.

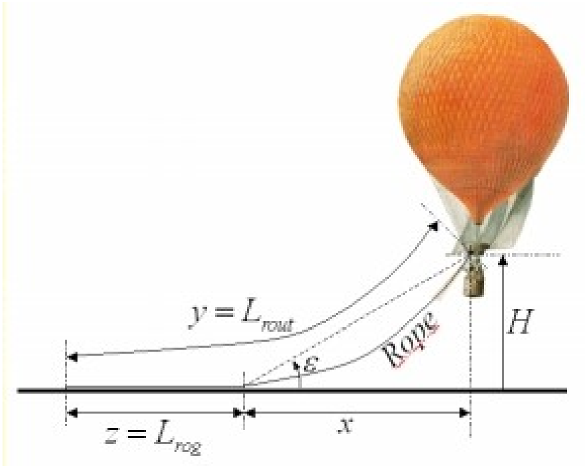
<u>Note</u>: The diagram above shows that M_{ztab} is very close to linear in both $\left|\omega_z\right|$ and V_w. The following can be extracted:

$$M_{ztab} \approx \left|\omega_z\right| \cdot \left(3 + 10 \cdot V_w\right) \cdot 10^4$$

and consequently

$$M_{zdamp} \approx -\omega_z C_F \left(3 + 10 \cdot V_w\right) \cdot 10^4$$

8. ***Appendix 2:*** *Algorithms for balloon drag rope*



Calculation of L_{rog}

$$(y - H) \geq z \geq \left\{ y + \mu H - \left[H^2 (1 + \mu^2) + 2\mu yH \right]^{1/2} \right\}$$

where μ is friction coefficient.
Ref.: Faxén, "Mechanics, Statics", Problem 26.15.

Case 1: "Balloon not moving, wind = 0".
$z_1 = y - H \Rightarrow x = 0$
The rope length z is in this case supposed to be curled up right below the balloon.

Case 2: "Balloon not moving, wind > 0".

$$z = y + \mu_2 H - \left[H^2 (1 + \mu_2^{\,2}) + 2\mu_2 yH \right]^{1/2}$$

$$\begin{cases} F_{aero} = z(\mu_2)\,\mu_2 \\ F_{aero} \approx F_{xAdyn} \end{cases}$$

μ_2 and thereby z is determined by iteration
The rope length is stretched out to windward.

Case 3: "Balloon moving".

$$z = y + \mu H - \left[H^2 (1 + \mu^2) + 2\mu yH \right]^{1/2}$$

The rope length is stretched out to windward, or - in other words – the rope is dragged behind.

Calculation of ε
Case 1: $\varepsilon = 90/57.3$

Case 2 and 3: $\varepsilon = \arcsin\left(\dfrac{H}{L_{rout} - L_{rog}}\right)$